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Breathing by the numbers

Outline

1. Generalities: trajectory.
   - Mathematics, geometry especially.
   - Two programs: Descartes & Newton; Beeckman, Mersenne, Boyle.
   - Mechanism & mechanisms (Anstey, Chalmers, Pyle).
   - Derived properties; *res vivens* in particular.
   - Patchworks and façades (M. Wilson).
2. The phenomena of the science of life.
   - Complexity; "organism" (Leibniz *apud* Smith).
   - Quantification: estimates of volume and pressure in natural philosophy; fluid mechanics *avant la lettre* (Bertoloni Meli).
3. "Geometrization" of *res vivens*.
   - Example: Steno and Borelli on the muscles (Bertoloni Meli).
   - Measures of force in living things.
4. An extended case: respiration.
   - (i) The movement of the lungs (Boyle, Borelli).
   - (ii) Pressure quantified (Pitcairn, Keill).
5. Return to generalities:
   - (i) Recognition of the special character of living things; but the extension of mechanism to derived properties permits a kind of subordination of the science of life to general physics
   - (ii) Success of the 2nd program of mathematization—discovery of tractable intensive quantities.
   - (ii) On the other hand, *organism* cannot be encompassed even within an enlarged mechanism.

Ad 1:

The "engineering" style
- No crossing of levels
- Idealizations in the interest of calculatory tractability
- Aim is to estimate quantities (e.g. volumes, flows)
- Partly for the sake of "how possible" explanations
- Desideratum: experimental measurements of estimated quantities

The "first principles" style
- Reduction from one level to another
  - The models used are not approximating idealizations; not constructed to aid calculation
- Only interactions within the microscopic level are explanatory (because visibly subordinated to universal laws of nature)
  - See Smith on "microscopic" analysis (Malpighi)
- The link between the invisible or microscopic and the visible or macroscopic is by way of *globular* effects, which only admit of human intervention

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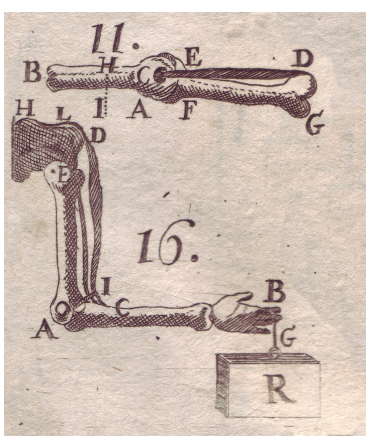
1. Generalities

My topic is respiration, or rather theories of its nature and function. I will use it to tie together several themes lately of interest to me. The first is obvious, given our occasion. We are to bring together mathematics and medicine in early modern natural history. Although there won't be much in the way of explicit mathematics in what follows, one theme of the paper as a whole is the applicability of mathematics, not as a timeless philosophical question, but as an evolving series of questions for natural philosophers engaged in attempting to understand, within the frameworks of their new sciences, the qualities and acts of living things.

The spirit in which I raise the issue owes something to recent work of Mark Wilson and Robert Batterman. Using examples from nineteenth- and twentieth-century physics, they have shown that the relation to the phenomena of the mathematical structures brought to bear on them in optics and aerodynamics doesn't exhibit the clean, uniform features one would expect from reading philosophers' treatments of axiomatized versions of classical mechanics or quantum theory. Instead the application of mathematical theories, themselves by no means smooth, to yield quantitative results across some physical domain results in a "patchwork", with one technique used in one region of the problem space and another elsewhere. In the intermediate realms of optics and turbulence there is no unified theory which would admit of reduction to first principles.

In the later seventeenth century *mathematics* meant *geometry*, first of all, under which I include the new calculus of Leibniz and Newton, but also *arithmetic*, including the theory of proportions and what we now call combinatorics. The authors I discuss below use only fairly simple tools from arithmetic and Euclid's geometry. Arithmetic has an obvious role in the calculation of volumes and the like: all one needs to apply arithmetic is quantities; the primary difficulty in its application to the life sciences is in getting nature to speak in numbers—a language which, despite Galileo, it does not readily converse in, not at least in such a way as to make the use of arithmetic or algebra fruitful. Geometry, for its part, is more demanding. Easy enough to hold that body is *res extensa*, identical with the objects of geometry; but where are the triangles and circles to which to apply the theorems of Euclid? It is striking that in the study of living things the most successful applications are in just those cases where carefully selected parts of an animal have the rigid components and simple relative mobility of the simple machines of mechanics.



From Borelli, *De Motu animalium*, plate 2 figure 11.

The mention of machines and mechanics brings me to my second theme. Alan Chalmers has argued that the "mechanical philosophy" made no real contribution to Boyle's experimental work; Antonio Clericuzio had earlier argued that Boyle's chemistry was not "subordinated" to his version of the mechanical philosophy. Andrew Pyle, Peter Anstey, and Bill Newman responded, defending the role of the mechanical philosophy. The crux of the dispute, at least for present purposes, is the scope of the mechanical philosophy, and in particular the place of "derived qualities" like elasticity and attraction. By "derived" I mean that in what one might call High Church or Cartesian mechanism such properties cannot be fundamental. The fundamental properties of bodies are restricted to a very short list: for Descartes, the modes of extension—of which he recognizes three categories, figure, size, and motion—to which many philosophers in the next generation found it necessary to add impenetrability, as not being, Descartes' efforts nothwithstanding, derivable from the modes of extension.

Low Church mechanism admits as basic, *provisionally* or *definitively*, qualities like the spring that Boyle attributed to the air or the attractive force held by Newtonians to inhere in all bodies simply by virtue of their being bodies. Nevertheless it prefers to keep the list as short as it can be consistently with explaining the phenomena, and so directs the philosopher to seek reductions where possible; it issues a peremptory refusal to some kinds of quality, notably those associated with the senses and also the "occult" powers of sympathy and antipathy; and finally it imposes a burden of proof on whoever would add a new quality to the list.

Mechanism so understood still contrasts with the Scholastic philosophy. The term remained current in the self-conception of natural philosophers partly for that reason. On the other hand, Clericuzio's notion of *subordination* is worth keeping in mind. In my view one of the developing stories of natural philosophy after the first generation is the increasing importance and independence of studies of derived qualities, studies that would grow into distinct branches of physics: fluid dynamics, optics, materials science, magnetism, electricity, chemistry, and so on. The practitioners of "special physics" may or may not expect or desire an eventual reduction of the qualities and quantities they study to something more fundamental; in practice, they proceed independently. Newtonian gravity is a case in point: even if the inverse-square attractive force were explicable in terms of vortices, as Fontenelle attempted to do, still nothing in the *Principia* depends on the availability of such an explanation. Special physics is in that sense not subordinate to general physics, i.e. to the science of the properties and laws pertaining to body as such.

The suggestion of a distinction between special and general physics will usher in my third theme. I want to distinguish, in a way that parallels but does not coincide with the distinction just made, between two styles of application of mathematics to the study of nature. The first I call the "engineering" style. I characterize it as follows:

(i) Its aim is to make quantitative estimates in support of "how possible" explanations (or, sometimes, "how not possible"), where the question is typically one that can be put in terms of design. So when Borelli, for example, wants to show that underwater travel is possible, or flight, he does so by designing a submarine or wings, and calculating the capacities of his setups. Having exhibited a *possible* mechanism, one moves to the *actual* by eliminating alternatives.

(ii) It idealizes, but for the sake of calculability, not in order to explain the phenomena in terms of the actions of entities at a more fundamental level than the level of the phenomena themselves. One replaces the irregularly shaped bones of the arm with rigid rectilinear beams, for example, not in the belief that our bumpy bones are, at some more fundamental level, smooth and straight, but because one believes or hopes that the quantities calculated using the ideal setup won't be too far off from the quantities realized by actual human bones and muscles.

(iii) It issues in, as desiderata, requests to the experimenter to generate quantitative (or, as the case may be, qualitative) results by which to test and perfect its models. One supposes, for example, that the action of the lungs consists in mixing air into the blood: and so one blows air into a pair of lungs while pumping dark venous blood into the pulmonary artery: the blood comes out bright and red, looking in other words like arterial blood looks *in vivo*. The demand for *measurement* issues, as I present it here, from theory; but it is plausible to suppose that in time a general demand to discovery new measurable features of things will arise, independently of particular hypotheses.

So much for the engineering style of applying mathematics to the study of nature. The other I call "first principles" style. It presupposes a contrast between levels, one of which is the visible, gross, or macroscopic, and the other of which is the invisible, subtle, or microscopic. The microscopic is basic in the sense that irreducible entities are to be found only at that level, and regress-ending explanations refer only to the interactions of those entities. So, for example, in explaining the action of an animal-machine that pulls its hand from a flame, Descartes reduces the movements of the animal to those of its animal spirits, that is to interactions of subtle matter which themselves require no further explanation to be intelligible.

Actions at the microscopic level are alone explanatory because they alone are intelligibly subordinated to the laws of nature. The applicability of geometry to natural things, on this view, requires that those things be, precisely, geometric; since visible, gross bodies are not—they are too complicated, they instantiate an excessively large catalogue of qualities—, a mathematical, that is a geometrical physics can hold immediately only of things at the microscopic level.

The models constructed in first-principles style applications of mathematics are not idealizations for the sake of calculability; they are supposed to show how things are at the microscopic level. The spherical second-element particles Descartes appeals to in his theory of the generation of colors by oblique collisions are not spherical so as to make the calculation of their paths more tractable; they are spherical *tout court*.

The relation between the microscopic-invisible and the macroscopic-visible (vision standing in here, as usual, for all the senses) is, in many cases, *globular*. The downward pull or resistance I feel in raising my arm—slowly—results from the action of innumerable corpuscles pressing it down; weight is a *globular* effect (we might say "statistical"), and as such geometrically intractable. Descartes' derivation of the macroscopic phenomena of the rainbow on the basis of the microscopic actions of light particles depended upon his being able to treat the trajectories (or trajectory-tendencies) of each particle independently of all the rest. In the case of weight or the effects of the animal spirits on the pineal gland no such assumption can be made.

I should note here that the term "microscopic" is mildly analogic. Justin Smith has noted that for Malpighi and, following him, Leibniz there is a literally microscopic level—a level of things visible not to the unaided eye but to the instrumentalized eye of the microscopist—in terms of which certain phenomena, e.g. those involving capillaries, can be re-described and explained. When I refer here to the microscopic, I have in mind not the Malpighian but the more familiar (to philosophers, at least) distinction between things available to us by sense, with or without the aid of instruments, and things whose existence is known to us only through their globular effects. The implied boundary between macro and micro is clearly movable if one admits instrumental observation, and in a world like Leibniz's, which has no absolute physical unit of length, and in which every body is indefinitely divisible, it cannot be set other than arbitrarily or *pro tempore*.

This, then, is the framework within which I will examine the works of some late seventeenth- and early eighteenth-century authors on respiration. Perhaps surprisingly, the engineering style and the first-principles style sit easily together in these works. As an immediate consequence one may infer that even in so "geometrical" an author as Borelli in his treatment of muscles and respiration, the presence of the first-principles style of reasoning does not require us to suppose that physiology is to be subordinated or reduced to physics; the engineering-style reasonings are quite independent of claims about the microscopic level; but neither does that independence show that the first-principles, which is to say the mechanical philosophy, can be dispensed with.

[Road map:

(i) Wotton on the role of mathematics in modern natural philosophy.

(ii) Arguments of Borelli on the action of the lungs in respiration and on the use of respiration.

(iii) Arguments of Keill.

(iv) Concluding remarks.]

2. Hors d'œuvre: ancients versus moderns

Among the many contributions to the quarrel of the ancients and moderns was the *Reflections upon ancient and modern learning* of William Wotton. This was written, it seems, at the behest of members of the Royal Society and published in 1694, at around the time Jonathan Swift was beginning to compose the *Tale of a Tub*, a work which Wotton was later to lambast as a satire on religion. The *Reflections* offer a lengthy, item-by-item critique of William Temple's *Essay upon ancient and modern learning,* itself a response to Fontenelle's *Digressions sur les anciens et modernes* (1688).

One point of comparison, on which Temple himself admitted the weakness of his case, was natural philosophy. Wotton, confining himself to the last eighty years, lays out a general case for the superiority of the Moderns, resting mostly on the "*modern Methods of Philosophizing*" (342).[1] The four points of the modern methods are familiar enough: that "Matter of Fact is the only thing appealed to", as the sole authority in settling disputes, and — more importantly here — that

1. I use the 1705 edition, which includes an essay by Richard Bentley and Wotton's *Defense of the Reflections*.

> *Mathematics* are joined along with *Physiology*, not only as Helps to Men's Understandings, and Quickeners of their Parts, but as absolutely necessary to the comprehending of the Oeconomy of Nature, in all her Work (343).

Moreover, in good Baconian fashion, the new philosophers "avoid making general Conclusions, till they have collected a great Number of Experiments or Observations upon the Thing in hand", letting refuted hypotheses "fall without any Noise or Stir". It is not that the Ancients took no notice of such maxims, but that they failed to put them into practice, that gives to the natural philosophy of the moderns its advantage.

What is striking in the comparison that follows is the prominence given by Wotton to the science of life (which he places among the "Physical Sciences", 346).[2] Galen, he notes, in giving an account of vision in *De Usu partium* (liv10c12), "makes a long Excuse" for the employment of a few simple geometrical terms. Now, in 1694, people believe "upon Trust", even if they have no mathematics, that "Geometry is of infinite Use to a Philosopher". Galen's readers would surely have been flummoxed by the mathematics of the moderns.

> If Three or Four Mathematical Terms were so affrightning, how would those learned Discourses of *Steno* and *Croone*, concerning Muscular Motion, have moved them? How much would they have been amazed at such minute Calculations of the Motive-strength of all the Muscles in the several general sorts of Animals, as require great Skill in Geometry, even to understand them, which are made by
> 1 *Borellus*, in his Discourses *of the Motion of Animals*? (347).

The method of the moderns was first adumbrated in the works of Bacon. Descartes, though "he was for doing too great a part of his Work in the Closet", nevertheless "to a vast Genius" joined

2. Glanvill's *Plus ultra*, from which Wotton draws much of his argument, devotes its second chapter to the advancements of "Chymistry and Anatomy".

"exquisite Skill in Geometry", and by the marriage of Geometry and Physics, "put the World in Hopes of a Masculine Off-spring in process of time, though the first Productions should prove abortive" (348). Those hopes were fulfilled with the rise of the Royal Society, among whose members Wotton mentions Boyle, Barrow, Newton, Huygens, Malpighius, Leeuwenhoek, Willoughby,[3] and Willis (349). Historians, including two here present, have recently argued that the life sciences, including physiology and medicine (considered both as a branch of natural philosophy and as a distinct discipline) deserve an equal place with physics and astronomy in the history of early modern science; Wotton does so, it would seem, as a matter of course.

The justice of Wotton's comparison need not detain us, nor the accuracy of his diagnosis of the shortcomings of the ancients, according to which a low opinion of the "Mechanical Arts" led philosophers to confine themselves to "those Studies which required few Hands and Mechanical tools to compleat them" (345; see Glanvill 27). What I want to take from Wotton's argument, which is hardly original (though it was controversial, eliciting a response from John Keill, brother of the James Keill to be studied later in this paper)[4] is that his chief examples of the use of geometry in natural philosophy are drawn from physiology, from the myology of Steno, Croon, and Borelli (in this respect he departs from Glanvill). Especially impressive to Wotton are the calculations of the last author in the *De Motu*, to which I now turn.

3. A botanist; author, with Ezeral Tonge, of *Some observations, directions and inquiries concerning the motion of sap in trees*; observations made with John Ray (*Roy. Soc. London Trans.* 48).

4. John Keill's response; Wotton's *Defense*.

3. Borelli

Borelli's *De motu animalium* was published posthumously in 1680. It draws on, but departs importantly from, results arrived at in collaboration with Malpighi. Malpighi's *Epistolæ de pulmonibus* (1661) had proposed that the function of respiration is
2 to refrigerate the blood; Borelli's work definitively rejects that position and, as we shall see, proposes a quite different function. Malpighi, who survived Borelli, recorded in his autobiography of 1697 his anger at having been publicly opposed by his former collaborator, and refutes Borell's arguments against him (see D. Bertoloni Meli's book).

The motus *of respiration*

Borelli considers first the *motus* or movement of respiration, then its *usus* or function. He begins with a list of the *phænomena* "that are observed in the movement of respiration" (pr81; 102). Air is taken in through the nostrils and mouth; in breathing, unlike eating, what goes in comes out through the same orifices. When a person breathes in, the ribs are spread slightly and elevated toward the collarbone along with the sternum, enlarging the chest.

Borelli describes a device by which to measure the increase in size. He takes a glass tube of known volume, and applying his mouth to one end withdraws from it as much air as would be inhaled in one light breath; a film of soapy water at the other end which, rising into the tube, yields a trace from which he can calculate the change in the length of the column of air in the tube, and thus also of the volume of air inhaled, and from this finally the increment of volume of the chest. From this experiment, he writes,

> I deduced that the volume of the air breathed in by me was not exactly 14 cubic digits; but suppose it was 15; now because the breadth of my chest, or its diameter, is not quite 15 digits, the approximate volume of my spheroidal chest when compressed would be equal to 3375 cubic digits; but after breathing in, 15 more cubic digits of air were added, and the augmented volume was 3390 cubic digits, the cube root of which is $15^1/_{50}$: therefore the entry of inhaled air increased the diameter of my chest not more than one fiftieth part of the thickness of a digit. From this I perceived that in light breathing the motion of the chest ought to be hard to discern, since we can notice only a slight elevation of the sternum toward the jugular (104).

One recognizes here the rhetoric of witnessing—the "I was there, I saw this" that confers authority on qualified perceivers; but also, and more significantly for my purposes, Borelli is transferring the measurement of volume, hence of linear dimension also, from the glass tube where it is tractable to the chest where it is not easily discerned. This bit of technique often passes without notice, and yet it implies a style of thinking, of overcoming momentary obstacles in the accumulation of facts by the application of ingenuity.

The use of mathematics here is at a minimum, though in the seventeenth century the taking of cubic roots required effort. Moreover, the relation of linear dimension to volume that Borelli is presupposing here was a commonplace. The mathematics in Borelli's work is not, even by the standards of his own time, very sophisticated. His knowledge did not extend much beyond Euclid's geometry.[5]

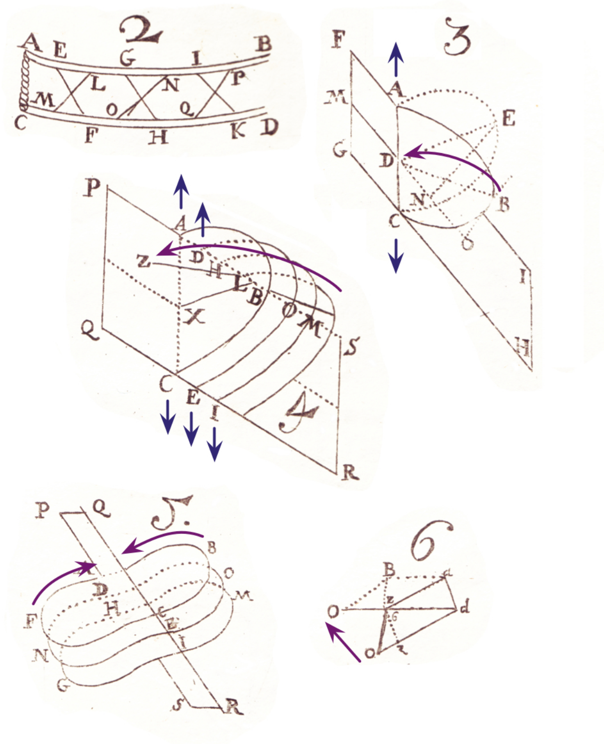
One should not, however, conclude from the rudimentariness of the mathematics applied by a natural philosopher to the rudimentariness of the thought involved in its application. I will

5. DSB; other sources. ✖

instance here some theorems leading up to the main conclusion of the chapter on the motion of respiration. Borelli intends to show that (i) in inhalation, the intercostal muscles, the diaphragm, and the elasticity of the air comprise the total efficient cause and that (ii) in exhalation, the muscles are passive, and the expulsion of air results from their relaxation together with the pressure of air outside the body on the chest.

To this end Borelli shows that the motion of the ribcage brought about by the action proposed by him of the intercostal muscles will have the effect of increasing its volume; and that the position of the ribcage at the end of inhalation, being "violent", will, when the muscles cease to be in tension, settle downward to its resting position, thus decreasing the volume contained in it.

The theorems and the accompanying diagrams are a fine example of the geometric analysis of physical setups found, for example, in Descartes' treatment of the rainbow. Borelli's theorems, as is customary in a synthetic treatment, run through in reverse the steps of the analysis. In Figure 5 we see what amounts to a very schematic representation of the ribcage; in Figure 4 we see one side of the ribcage; finally in Figure 3 (which is of course the *first* figure the reader encounters) we see a single rib and (in the printed text that refers to this Figure) the supposed motion of that rib, abstracted from its fellow ribs and the rest of the body, during inhalation.



From Borelli *De motu* plate 18, figures 2-6. Arrows added to show the motion imputed to elements of the figures. NB. In Figure 2 the slanting lines represent a putative configuration of the intercostal muscles.

I will mention just one of the theorems proved by Borelli. Its proof refers to Figure 4 above. The theorem states that

> If the extremities A, D, H in Figure 4 of Plate 18 are affixed to the immobile column PS and the extremities C, E, I to the movable line QR, and the planes of the arcs, which are equidistant from one another, make acute angles with the plane PR which contains their termini, and if the apexes B, O, M of the arcs are pulled in the direction from M to B parallel to the plane PR so that [the planes of the arcs] make obtuse angles with PR—in other words, if they are pulled toward Z—I say that the semicylindrical cavity ABCIML is broadened [*amplior efficietur*], and that when the pulling stops it will spontaneously revert to its earlier narrow form (110).

I won't go through the proof except to say that it involves noting that if the arcs move as described the apexes must move closer to the plane PR containing their extremities, thereby forcing those extremities apart, as the ends of a bow are forced apart when the bow is drawn.

Having proved this claim about the change in volume of the chest cavity when the intercostal muscles are in tension, Borelli infers that their motion, along with that of the diaphragm, is sufficient to bring air into the lungs (111–112), and that in exhalation all that is required is that those muscles relax.

Thus does he answer one of the major questions in the theory of respiration. The argument is in what I called the "engineering" style. Borelli both shows that his mechanism will do the job and eliminates alternatives, e.g. the mechanism in Figure 2 above, in which one set of muscles pulls the ribs together and another pulls them apart. Though in the background perhaps there is a rejection of theories attributing the inrush of air into the lungs to some sort of attractive power in them (or to the *horror vacui*, which he explicitly rejects), Borelli's explanation, though "mechanical", does not consist in postulating microscopic entities to whose (insensible) actions the tangible, if not visible, motions of the air or the muscles are reduced. There is just one level; but that level undergoes a succession of extractions—that is, of parts or subassemblies from the complicated whole—and simplifications, which we can to some extent see in the diagrams, until an object is attained to which geometric reasoning can be applied.

The usus *of respiration*

Having demonstrated how the organs of respiration operate, Borelli turns to their function or *usus*—a traditional division in medical texts. Nature, he says, is "accustomed to obtaining, by a unique action and a single instrument, several useful things". The *primary* use of respiration is the conservation of life; what remains to be seen is "how such a notable good is produced, and which mechanical actions, consequent upon this aim, are used" (118).

That use, as it turns out, is *proximately* to mix together minimal particles (*minimas particulas*) of air with minimal particles of water (138, pr113), and *remotely* to bring about oscillatory motions by which to regulate the functions of the animal body (143). From the mention of *minima*, one might guess that the *usus* of respiration is to be explained in "first principles" style—and it is. To explain the role of air in animal life Borelli, like Descartes, adverts to bodies whose efficacious features—in the present instance, their oscillatory motions—cannot be apprehended by the senses (which *consist* in part of bodies of that very same sort).

Borelli opposes the view according to which the air in the lungs passes directly to the blood. Instead the function of the lungs is to reduce to homogeneous *minima* the heterogeneous particles of the blood entering them by crushing and contrary movement in the tiny ramified passages of the lungs—those which Malpighi, Borelli's sometime collaborator, was the first to observe. Borelli argues:

> […] exact mixture [*miscelam*] of the *blood* must occur in the *lungs*. But because exact mixing [*mistio*] cannot occur unless each minimal particle of one nature touches minimal particles of diverse constitution, so it is that in the *lungs* mixing by minima must occur. But without contrary vertiginous motions and without repeated crushing particles
> 3 of one nature cannot insert themselves, in the manner of wedges, between particles of another [nature]. Hence in the *lungs* such contrary motions and crushings of the *blood* must

> occur. This cannot occur unless in places that are wide like sacks or bags closed on all sides […] Therefore the cavity of the *lungs* would have to be such, which is false […] (131, pr108).

It follows that

> because the *vessels of the lungs* are very similar to the tubes discussed above [see Figures 8–11 from Plate 18], and because in them mixture cannot occur by grinding [*ope contusionis*], as has been said [pr105]: therefore neither could the mixing [of blood and air] be completed in the *lungs*,

so that even though there is grinding of the blood in the lungs, nevertheless that operation does not yield mixture (131).

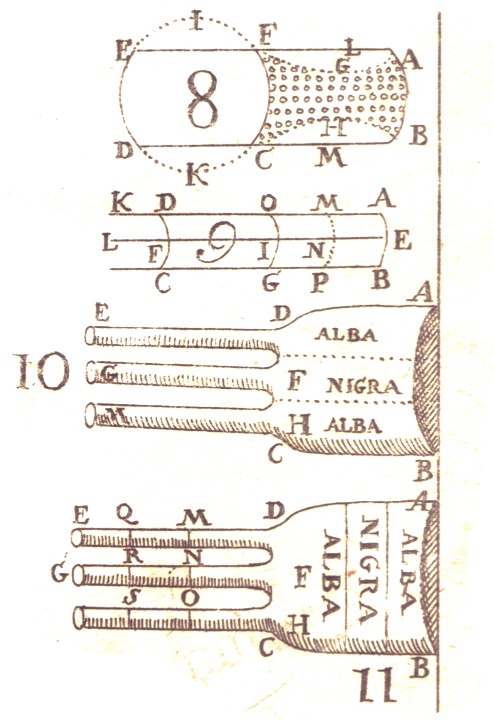
Borelli's model of mixing

In proving this claim Borelli appeals to simple geometrical features of flows, which he thinks will be obvious (Barbara Orland: a shift from "humoral" to "hydraulic" models of the body in medicine). But in order to reach the point of appealing to those features, he must first construct a model.

> As is customary in mechanical science [*scientia mechanica*], the subject-matter of a proposition must be abstracted from
> 4 material variations and circumstances; or conditions must be equal [*vel conditiones pares esse debent*]. And so in our case we suppose first of all two heaps of *grains of millet*, one of which is composed of white, the other of black granules. The round figures are the same [in shape] and equal [in size], and equally heavy (113).

As Bertoloni Meli observes, Borelli ignores color in his description of the phenomena to be explained. Abstracting from color is therefore presupposed (Borelli may also be thinking of Aristotelian theories according to which colors consist in varying mixtures of black and white). Black and white suffice, moreover, because presumably the mixing of several liquids will proceed on the same principles as the mixing of two.

Mixing cannot occur except if particles of one sort, say white, wedge themselves between particles of the other, which must therefore move laterally so as to let them in, and then obliquely so as to occupy the place occupied by the white particles which are now between black particles. Mixing is a kind of dance, in which the partners have assigned motions (124–125).



From Borelli *De motu* plate 18, figures 8–11.
Mixing of particles in tubes.

We come now to Figures 10 and 11 [in the handout]. Borelli claims that if a tube—in diagram the tube ABCD—is subdivided into others—DE, FG, HM—and if fluids enter, or white and black grains, even though they may be compressed they will not be mixed with one another. Combinatorially speaking, there are two possibilities. In Figure 10 the white and black enter in parallel, as we would say, and in Figure 11 they enter in series. It does seem that if they are so arranged, and if the flow is not turbulent, they will enter the smaller tubes without mixing. I'll leave it to you to poke holes in Borelli's reasoning; what matters here is not its soundness but the kind of reasoning it is.

A mathematician now might regard it as topological, in a broad sense—as belonging to what Leibniz called *analysis situs*. Seventeenth-century mathematicians, having no explicit topological concepts (Leibniz aside), would have called it geometric—again in a broad sense. The idea is so simple that one might overlook it altogether as an application of mathematics. But similar ideas were noted and appealed to. Beeckman, for example, in his inaugural address at Dordrecht in 1627, in arguing that his pupils will be well-educated and their parents' money therefore wisely spent, presents them with a series of applications of the "isoperimetric principle" and of scaling laws—for example, that the ratio of boundary length to surface area of similar figures is in inverse proportion to their linear dimension; from which he justifies, among others, the observation that bigger cities are easier to defend than smaller ones (because you can put more defenders per unit length at the perimeter).

The mathematics here is not, in fact, as trivial as it looks. But it requires a great deal of development, both of concepts and of standards of rigor, to see that it is not trivial. The truths appealed to by Borelli and Beeckman are simple to state (in their easiest cases) and evident (as long as you don't delve too deeply into the grounds upon which they might be rigorously proved). Although I tend to think of this sort of mathematics and its application as characteristic of the engineering style, reasonings appealing to it occur in both the engineering and the first principles styles. But in the latter they occur just insofar as the geometric models, themselves at a more fundamental level than the phenomena, are treated as if they were not relevantly different from their macroscopic analogues. In imitation of Bertoloni Meli's diagram of objects in seventeenth-century mechanics (✖) one might produce a diagram of objects in physiology, among which would be sacks, grains, and tubes, along with animals living and dead, surgical tools, figures like those of Borelli reproduced above, clocks, and so forth. [Resemblance of the "network of objects" to Wilson's version of classical physics.]

(The clock, as you might expect in this mechanist setting, does make an appearance, not as a generic machine, but as a device whose motions are regulated.

> For which reason, as in clocks, so in animals or automata of Nature a regulatory machine [*machina regulatrix*] must be adjoined, which by mechanical necessity restrains the motive force [*vim motivam*], so as not to transgress the laws instituted by the Divine Architect (143, pr116).

Here we "unravel [*detegimus*]", Borelli concludes, "the great mystery of the necessity of air in animals". The air mixed into the blood imparts to it and to other bodily fluids an oscillatory motion by which their more vivacious and vehement motions may be both stimulated and controlled (144).)

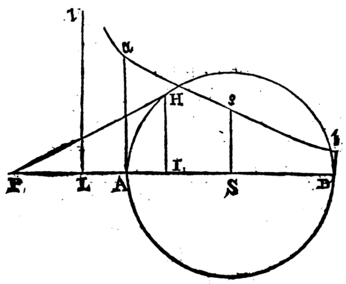
4. Keill

Between 1680, the date of the posthumous publication of Borelli's *De motu*, and 1708, the date of James Keill's *Animal secretion*, one innovation stands out: the successful introduction of the attractive force of gravity in Newton's *Principia* to explain certain motions of terrestrial and celestial bodies. Unlike earlier such forces, Newton's is regulated by its subordination to the law according to which it is inversely proportional to distance and directly proportional to the new physical quantity *mass*. Newton's success encouraged imitation. James Keill, whose brother was among the champions of Newtonianism, introduces, in *Animal Secretion*, the Newtonian postulation of forces into physiology. Bodily fluids like milk and urine, he observes, consist upon inspection under the microscope of "very small Globules, swimming in a limpid fluid" (5); and although we cannot likewise resolve blood into the various fluids which we know to be extracted from it by the Glands of body, it is reasonable to suppose that it too consists of globules in a clear fluid.

Unlike dissolved salts, the particles of which the globules are composed must, if those globules are to maintain their integrity, be more strongly attracted to each other than to the particles of the aqueous fluid surrounding them. There must be, says Keill (he credits his brother John with the discovery) an attractive power whose effect is to make those particles cohere. Upon this power the "whole Animal Œconomy" depends; "it seems to be the only Principle, from which there can be a satisfactory Solution given of the *Phænomena*, produc'd by the *Minima Naturæ*" (8) in animal bodies. That power must be proportional, not, as gravity is, to the square, but to some higher power, of the distance between particles. Unlike gravity, therefore, the force with which one body attracts another will vary with shape, "according as the Particles are Cones, Cylinders, Cubes, or Spheres" (19); and on the basis, perhaps, of an isoperimetric principle, Keill holds that a "Spherical Particle has the strongest attractive Power".

[Figures on pp14–15: "If Particles of Matter attract each other with a Force, that is in a reciprocal triplicate, or a greater proportion of their distances, the Force by which a Corpuscle is drawn to a Body, made up of such attractive Particles, is infinitely greater at the Contact, or Extremely near it, than at any determined distance from it".]

14 *Of Animal Secretion.*

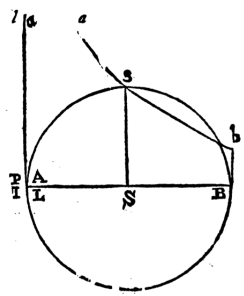


Suppoſe the Sphere A H B compoſed of Particles, that attract a Corpuſcle P with a Force reciprocally proportional to the Cubes of their Diſtances. Draw the Tangent P H, and from H let fall the perpendicular H I, biſect P I in L, and raiſe the Perpendiculars L l, A a, *Ss*, B b, and make S s = to S I: with the Aſymptots L B, L l thro' s, deſcribe the Hyperbola b B a, and the Area a A B b — the rect-

Of Animal Secretion. 15

angle 2 A S x S I will repreſent the Attraction of the Corpuſcle P by the 81 Prop. of Sir *Iſaac Newton*'s Principles.

But when the Corpuſcle P comes to the Sphere, and touches in A, then the Points P, L, A, I, and H, coincide, and A a becomes the Aſymptot of the Hyperbola, and the Area a A B b a becomes infinite, and the rectangle 2 A S x S I being finite, the Area a A B b a — 2 A S x S I will be infinite; and conſequently the Force, by which the Corpuſcle P is attracted by the Sphere, will be likewiſe Infinite.



Keill then proves a series of propositions or "laws" concerning such forces. He then takes up the various basic functions of the body, starting with respiration. Secretions, he holds, "are first formed in the Blood, before they are separated by the Glands". But the condition of the blood is not stable. It tends to coagulate, and the job of the lungs is divide it anew into *minima*. The problem Keill then addresses is whether the force of respiration suffices to do this.

> The Particles of the Blood returning by the Veins mutually attract one another, and cohering form Globules too big for ay Secretion; and therefore there was an absolute necessity, that they should be broken and divided in the Lungs by the force of Respiration: which because it is commonly thought to be inconsiderable, by reason we are not sensible of it, I shall therefore here make an Estimate of it.

To this end he adduces a proposition from hydrostatics: the forces that propels the same quantity of fluid from unequal tubes are in proportion to the squares of the times and of the radii of the tubes [check this]. Using this principle, Keill calculates the force by which the air is "thrust out of the Lungs in Expiriation".

He fills up a hog's-bladder, whose volume he knows to be equal to that of one outward breath, with air, and affixes a tube to its neck. Having filled the bladder with air he finds that a weight of 2 pounds 4 ounces (= 36 ounces) forces out all the air "in the space of 25 Vibrations of a Pendulum, which vibrated Seconds of a Minute" (26). With the required quantities in hand, he calculates the force with which the air is breathed out to be 1600 ounces or 100 pounds; and because "Action and Reaction are equal, the Pressure of the Air upon the Lungs every Expiration is equal to the Pressure of an 100 *lb* Weight" (27).

I will not vouch for the correctness of the reasoning. Keill, applying it to the question raised earlier, concludes that

> Nobody doubts whether this Pressure of the Air upon the Lungs in breathing be sufficient to break the Globules of the Blood, and to dissolve all the Cohesions they might contract in their Circulation thro' the Arteries and Veins.

The blood, "thus dissolved", is "thrown out by the Heart into the *Aorta*", and the particles in it will begin to recombine "according to their several attractive Powers".

[▶ Keill returns to that point several times. In explaining, for example, why the blood follows such a long path to the liver, he notes that bile, the liver's primary secretion, is produced in very small quantities. "In a large Dog, whose *Ductus Cholidochus* was near as big as a Man's, I could never gather above two Drachms in an hour" (47). Yet the liver receives 500 ounces of blood per hour, and so the proportion of bile to blood is "at least, as one to two thousand". In order to reduce the volume of the incoming blood so greatly, almost all the other substances contained in it must first be removed, and that can occur only if the flow of the blood is slowed down sufficiently to let the particles of those other substances cohere and drop out of it (46). The "intent of Nature", in designing the mesenteric circulation, was to "diminish the Velocity of the Blood", so that finally the liver might remove its Bile.]

Like Borelli, Keill makes a point of *estimating quantities*, for which purpose he also sometimes *measures* them: the measuring, in this case, is driven by the estimating. Estimation requires, in addition to physical principles, the application of mathematics. Again, the mathematics is very simple, amounting to no more than arithmetical manipulations. But it is no less essential for all that. As Keill himself puts it in his preface,

> Tho' any one with a moderate Skill in the Mathematicks may understand these Discourses, yet without that no one can judge of their Truth, and Usefulness in explaining the Animal Oeconomy (xxviii).[6]

5. Conclusion

6. Borelli quoting Plato: *ageometrici…*