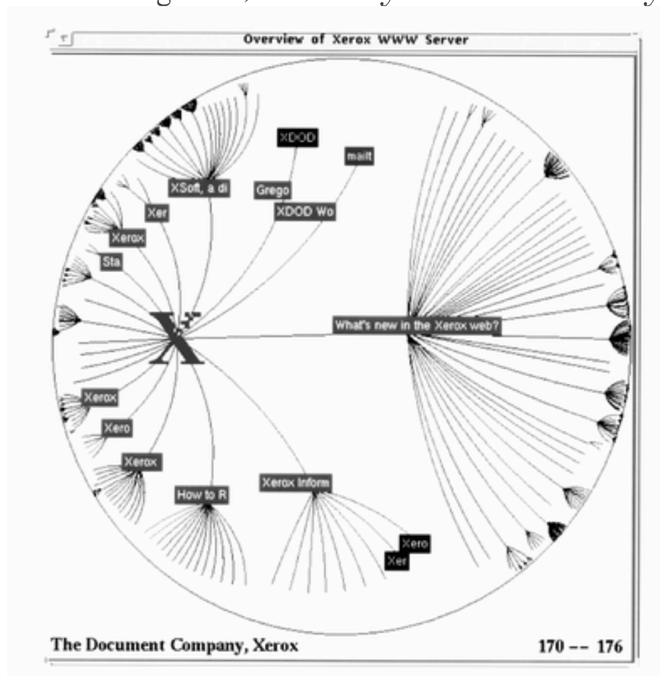
Lynxes and lumps

I've been reading Robert Batterman's *Devil in the details*, a book that packs a lot of punch in a relatively few pages. Among its themes is that of the *universality* of certain mathematical models. Universality is "the slightly pretentious way in which physicists denote identical behaviour in different systems" (Berry 1987:185, quoted in Batterman 13).

That requires some unpacking. Two systems exhibit "identical" behavior if that behavior can, under suitable redescription, be seen to instantiate the same mathematical system (I use the imprecise word 'system' rather than a more precise term because what is instantiated need not be, for example, the graph of a single equation). They are different if, as in the case of Berry's own examples, they have different shapes, or if, as in some cases discussed by Batterman, they are made of different stuffs. We will see yet another sort of difference below.

Let me start instead with something simple: the directed graph or *digraph*. Family trees and citation networks instantiate that structure: draw an arrow from *x* to *y* if *x* is a progenitor of *y* or if *y* is cited by *x*. More interestingly, so-called "scale-free" networks, though arising in different real-world situations (different in the sense of being realized on quite different scales by quite different sorts of process), obey the same *statistics* (for example, the number of arrows entering a node—think of links *to* a site—obeys a power-law distribution): the probability of a node's having *n* entering links is inversely proportional to some small power *n k* of *n*. Many nodes will have only a few entering links, and a very few will have many.



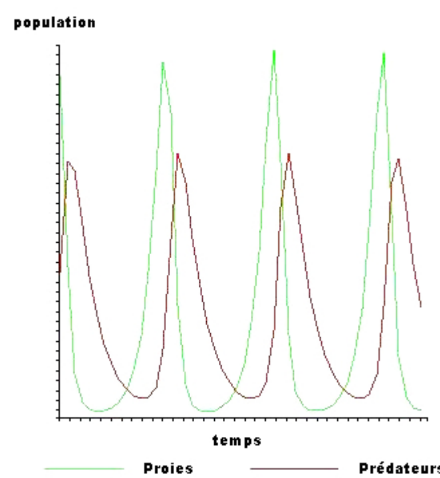
Source: J. Lamping, R. Rao. "The hyperbolic browser: a focus+context technique for visualizing large hierarchies". *Journal of Visual Languages and Computing* 7 (1996) 33–55, fig. 14. See also here.

I would prefer to call the phenomenon "generality". Not all networks, let alone all the things that can be modelled by digraphs, obey power-law distributions in the distribution of links; but those that do are expected to exhibit other similarities as well—for example, to have arisen by a "rich get richer" process wherein nodes that already have many entering links are more likely to receive new entering links than nodes that have just a few entering links. Were it true that scale-free networks could arise *only* by such processes, we might know this quite independently of knowing the physical means by which links are made, or the causes that lead, for example, one blogger to link to others. Scale-free networks or (equivalently, under the hypothesis just mentioned) "rich get richer" networks would be a *genus* of network, to which the mathematics of one kind of mathematical structure applied, and whose formation occurred by a process to which again the mathematics of one kind of structure applied. Not only the network structure but the process of its formation could be described independently of the stuffs and causal processes required in any instantiation of the structure. Universality or generality, so understood, offers, in Batterman's view, a promising way to think about, among other topics, multiple realizability and emergence.

Source: Wikimedia;
originally posted to Flickr as
Lynx Canadensis by Keith
Williamson, 21 March 2010.

Now for the lynxes and lumps. First the lynxes. Canadian lynxes live almost entirely on a diet of snowshoe hares, and snowshoe hares are preyed on almost exclusively by lynxes (or so the story goes: for complications see Finerty 1979 and Ranta et al 1997). The rate of growth in the population of the hares depends not only on endogenous factors like fertility but also on the likelihood of fatal encounters with lynxes; on the other hand, the rate of growth in the population of the lynxes depends on the likelihood of those same encounters together with endogenous factors (including emigration from one region to another). Sadly it seems that this is not in fact a classic predator-prey relation: see Zhang et al 2007; but, being philosophers, we'll ignore, as the BBC and other textbooks do, that embarassing truth.

The mathematics of the two-species case is well understood. Alfred James Lotka in 1925 and Vito Volterra in 1926 solved what are now called the Lotka-Volterra equations. These yield periodic fluctuations of the predator and prey populations that are out of phase by a fixed amount. (For three species, see He et al 2011.)



Source: wikipedia.fr.

Real life is messier: the phase difference doesn't seem to be constant, for example. Other factors, as I've mentioned, may intervene—the effect of large-scale climate oscillations, in particular El Niño/La Niña, on the variation in population is considered by Zhang et al; in this case it turns out to be insignificant.

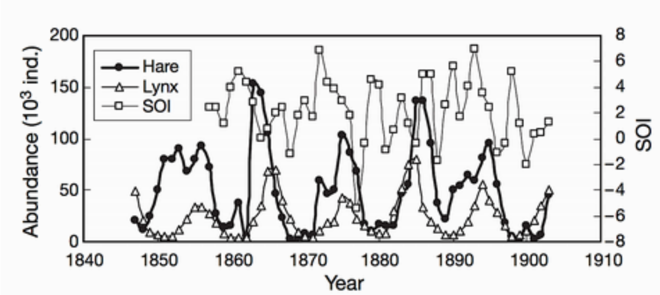


Fig. 1. *Lepus americanus* and *Lynx canadensis*. Hare and lynx abundances (redrawn from Leigh 1968) and Southern Oscillation Index (SOI) (data from Shi & Wang 1989) during 1847–1903

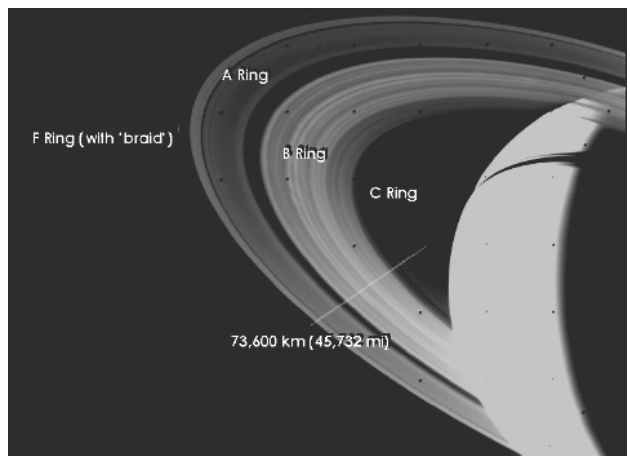
Source: Zhang et al 2007:85.

And the lumps? The lumps in question inhabit the rings of Saturn, specifically the F ring. It has been proposed that

> the collective behavior of the ring particles resembles a predator–prey system: the mean aggregate size is the prey, which feeds the velocity dispersion; conversely, increasing dispersion breaks up the aggregates. Moons may trigger clumping by streamline crowding, which reduces the relative velocity, leading to more aggregation and more clumping. Disaggregation may follow from disruptive collisions or tidal shedding as the clumps stir the relative velocity. For realistic values of the parameters this yields a limit cycle behavior, as for the ecology of foxes and hares or the "boom-bust" economic cycle (Esposito et al 2012:103).

(That reference to economics is no mere grace note. Among the many claimed applications of the Lotka-Volterra equations is an explanation of business cycles: see Wills 2010. Another application is to ideological struggles: see Ausloos et al 2011.

After an analysis of some of the factors contributing to the aggregation of particles, on the one hand, which is affected mainly by the mean mass, and their fragmentation, on the other, which is affected by the dispersion of velocities (particles in roughly the same orbit that collide are more likely to break up if their relative velocities are higher), the authors arrive at equations whose form, they note, resembles that of the Lotka-Volterra equations. They interpret their own to yield predators and prey:



Source: Ringclimber (NASA Voyager).

The mean aggregate mass corresponds to the prey population; the velocity dispersion corresponds to the predators: it 'feeds' off the accelerations from the aggregates' gravity. If the velocity dispersion grows too large, it limits the prey: higher velocities fragment the aggregates. In the absence of interaction between mass and velocity, the prey (mean aggregate mass) grows and the predator population (dispersion velocity) decays. When they interact, the ensemble reaches a stable equilibrium for size distribution and a corresponding thermal equilibrium (e.g., Stewart et al., 1984).

Note that both the “prey” and the “predator” are not things but attributes, and attributes not of things but of populations: the *mean* mass, the velocity *dispersion*. It is true that the mean mass would in principle be reducible to the masses of individual particles, and similarly for the velocity dispersion. But the model is not applied to the individuals. It is applied to those attributes.

We are presented here not with “multiple realizability” but with a more radical sort of generality. The lynxes are coordinate with velocities or velocity dispersions, the hares with mean masses. In fact even in the original application the quantities whose variation is determined by the equations are population numbers; and though one might insist that in principle the behavior of the system—the phase-locked fluctuations of those numbers—could by an omniscient observer be reduced to or derived from all the many interactions of individuals whose joint result at each moment is the population numbers, nevertheless the description of the lynx-hare system or of the aggregation-fragmentation process requires no reference to any such events except in the aggregate, and the manner in which they occur has a role only in making it plausible that the equations apply (see Esposito et al 108–110).

Call the feature of being an instantiation of the Lotka-Volterra model *LV*-ness. Is that an emergent property? Unlike some of Batterman’s examples, it does not involve, at least explicitly, asymptotic reasoning. A great many assumptions are made to arrive at a system simple enough to yield tractable equations, both about the objects in the system (the particles of which aggregates are composed are stipulated to have the same mass) and about the calculations (approximations simple in form replace more accurate but less convenient estimates, e.g. of the time between collisions for an arbitrary particle).

Here we enter territory well explored by Mark Wilson in *Wandering Significance*: the T_{coll} that enters into the eventual predator-prey model (and thus into the attribution of *LV*-ness to the F-ring system) stands in no simple relation to actual times between collisions of actual particles; or, to put it another way, the attaching to things in the world of that term, though not arbitrary or a mere construction, nevertheless veers quite far from the “classical” story of reference that Wilson drastically revises.

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See also Albers et al 2011 (pdf).

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